**What is an equation?**

A statement of equality of two algebraic expressions which involves one or more literals (variables) is called an equation.

3 + x = 7 is an equation.

The set of values of variables which makes the open sentence true is called the solution set.

**Note:**

Every equation has two sides — L.H.S. (left-hand side) and R.H.S. (right-hand side).

Literals involved in the equation are called variables. These are usually denoted by letters of English alphabet.

An equation may contain any number of variables.

**For example:**

(i) 5x + 7 = 19   (ii) 2x + 13y = 8   (iii) 5x - 3y + 4z - 14 = 0

**Find the solution set for the following open sentences.**

  (a) x + 4 = 7

  **(a) x + 4 = 7**

**Solution:**

x + 4 = 7

If x = 0, then 0 + 4 ≠ 7

If x= 1, then 1 + 4 ≠ 7

If x = 2, then 2 + 4 ≠ 7

If x = 3, then 3 + 4 = 7

Therefore, the solution set for the open sentence x + 4 = 7 is 3.

**What is a linear equation?**

An equation which involves only one variable whose highest power is 1 is known as a linear equation in that variable.

**For example:**

(a) x + 4 = 19

(b) y - 7 = 11

(c) x/2 - x/3 = 9

(d) 2x - 5 = x + 7

(e) x + 13 = 27

(f) y - 3 = 9

(g) 11x + 5 = x + 7

The sign of equality divides the equation into two sides. Left hand side or L.H.S. and Right hand side or R.H.S

Solution of linear equation or Root of linear equation: The value of the variable which makes left hand side equal to right hand side in the given equation is called the solution or the root of the equation.

**For example:**

**1.**x + 1 = 4

Here, L.H.S. is x + 1 and R.H.S. is 4

If we put x = 3, then L.H.S. is 3 + 1 which is equal to R.H.S.

Thus, the solution of the given linear equation is x = 3

**2.**5x - 2 = 3x - 4 is a linear equation.

If we put x = -1, then L.H.S. is 5 × -1 -2 and R.H.S. is 3 × -1 -4

                                            = -5 -2                           = -3 -4

                                            = -7                               = -7

So, L.H.S. = R.H.S.

Therefore, x = -1 is the solution for the equation 5x - 2 = 3x - 4

### How to solve linear equation in one variable?

**Rules for solving a linear equation in one variable:**

The equation remains unchanged if –

(a)*The same number is added to both sides of the equation.*

**For example:**

**1.**x - 4 = 7

⇒ x - 4 + 4 = 7 + 4 **(Add 4 to both sides)**

⇒ x = 11

**2.**x - 2 = 10

⇒ x - 2 + 2 = 10 + 2 **(Add 2 to both sides)**

⇒ x = 12

(b) *The same number is subtracted from both sides of the equation.*

**For example:**

**1.**x + 5 = 9

⇒ x + 5 - 5 = 9 - 5 **(Subtract 5 from both sides)**

⇒ x + 0 = 4

⇒ x = 4

**2.**x + 1/2 = 3

x + 1/2 - 1/2 = 3 - 1/2**(Subtract 1/2 from both sides)**

⇒ x = 3 - 1/2

⇒ x = (6 - 1)/2

⇒ x = 5/2

(c) *The same number is multiplied to both sides of the equation.*

**For example:**

**1.**x/2 = 5

⇒ x/2 × 2 = 5 × 2 **(Multiply 2 to both the sides)**

⇒ x = 10

**2.**x/5 = 15

⇒ x/5 × 5 = 15/5 **(Multiply 5 to both the sides)**

⇒ x = 3

(d) *The same non-zero number divides both sides of the equation.*

**For example:**

**1.**0.2x = 0.24

⇒ 0.2x/0.2 = 0.24/0.2 **(Divide both sides by 0.2)**

⇒ x = 0.12

**2.**5x = 10

⇒ 5x/5 = 10/5 **(Divide both sides by 2)**

⇒ x = 2

### What is transposition? Explain the methods of transposition.

Any term of an equation may be shifted to the other side with a change in its sign without affecting the equality. This process is called *transposition*.

So, by transposing a term —

● We simply change its sign and carry it to the other side of the equation.

● **‘+‘** sign of the term changes to **‘—‘** sign to the other side and vice-versa.

● **‘×’**sign of the factor changes to **‘÷‘** sign to the other side and vice-versa.

● Now, simplify L.H.S. such that each side contains just one term.

● Finally, simplify the equation to get the value of the variable.

**For example:**

**10x - 7 = 8x + 13**

⇒ 10x - 8x = 13 + 7

⇒ 2x = 20

⇒ 2x/2 = 20/2

⇒ x = 10

**Note:**

    + changes to –

    – changes to +

    × changes to ÷

    ÷ changes to ×

Therefore, from the above we came to know that without changing the equality, this process of changing sign is called transposition.

### How to solve linear equations?

Step-by-step instructions are given in the examples of solving linear equations. We will learn how to solve one variable linear equations using addition, subtraction, multiplication and division.

**Examples on solving linear equations:**

**1.** Solve the equation 2x - 1 = 14 - x and represent the solution graphically.

**Solution:**

2x - 1 = 14 - x

⇒ 2x + x = 14 + 1

**(Transfer -x from right hand side to the left hand side, then negative x changes to positive x. Similarly again transfer -1 from left hand side to the right hand side, then negative 1 change to positive 1.**

**Therefore, we arranged the variables in one side and the numbers in the other side.)**

⇒ 3x = 15

⇒ 3x/3 = 15/3 **(Divide both sides by 3)**

⇒ x = 5

Therefore, x = 5 is the solution of the given equation.

The solution may be represented graphically on the number line by graphing linear equations.

**2.**Solve the equation 10x = 5x + 1/2 and represent the solution graphically.

**Solution:**

10x = 5x + 1/2

⇒ 10x – 5x = 1/2

**(Transfer 5x from right hand side to the left hand side, then positive 5x changes to negative 5x).**

⇒ 5x = 1/2

⇒ 5x/5 = 1/2 ÷ 5 **(Divide both sides by 5)**

⇒ x = 1/2 × 1/5

⇒ x = 1/10

Therefore, x = 1/10 is the solution of the given equation.

**3.** Solve the equation 6(3x + 2) + 5(7x - 6) - 12x = 5(6x - 1) + 6(x - 3) and verify your answer

**Solution:**

6(3x + 2) + 5(7x - 6) - 12x = 5(6x - 1) + 6(x - 3)

⇒ 18x + 12 + 35x - 30 - 12x = 30x - 5 + 6x - 18

⇒ 18x + 35x - 12x + 12 - 30 = 30x + 6x - 5 - 18

⇒ 41x - 18 = 36x - 23

⇒ 41x - 36x = - 23 + 18

⇒ 5x = -5

⇒ x = -5/5

⇒ x = -1

**Therefore, x = -1 is the solution of the given equation.**

Now we will verify both the sides of the equation,

6(3x + 2) + 5(7x - 6) - 12x = 5(6x - 1) + 6(x - 3) are equal to each other;

**Verification:**

**L.H.S.** = 6(3x + 2) + 5(7x - 6) - 12x

Plug the value of x = -1 we get;

= 6[3 × (-1) + 2] + 5 [7 × (-1) - 6] - 12 × (-1)

= 6[-3 + 2] + 5[-7 - 6] + 12

= 6 × (-1) + 5 (-13) + 12

= - 6 - 65 + 12

= -71 + 12

= -59

**Verification:**

**R.H.S.** = 5(6x - 1) + 6(x - 3)

Plug the value of x = - 1, we get

= 5[6 × (-1) - 1] + 6[(-1) - 3]

= 5(-6 - 1) + 6(-1 -3)

= 5 × (-7) + 6 × (-4)

= - 35 - 24

= - 59

***Since, L.H.S. = R.H.S. hence verified.***

### What is cross multiplication?

The process of multiplying the numerator on the left hand side with the denominator on the right hand side and multiplying the denominator on left hand side with the numerator on right hand side is called cross multiplication.

And then equating both the products we get the linear equation.

On solving it we get the value of variable for which L.H.S. = R.H.S. Then, it is an equation of the form.

(mx + n)/(ox + p) = q/r where m, n, o, p, q, r are numbers and ox + p ≠ 0
⇒ r(mx + n) = q(ox + p)

It’s an equation in one variable x but it is not a linear equation as L.H.S. is not a linear polynomial.

We convert this into linear equation by the method of cross multiplication and further solve it step-by-step.

**Examples on cross multiplication while solving linear equations:**

**1.**(3x + 4)/5 = (2x - 3)/3

**Solution:**

(3x + 4)/5 = (2x - 3)/3

On cross multiplication, we get;

⇒ 3(3x + 4) = 5(2x - 3)

⇒ 9x + 12 = 10x - 15

⇒ 9x - 10x = -15 - 12

⇒ -x = -27

⇒ x = 27

**Verification:**

**L.H.S.** = (3x + 4)/5

Plug x = 27, we get;

(3 × 27 + 4)/5

= 81 + 4/5

= 85/5

= 17

**Verification:**

**R.H.S.** = (2x - 3)/3

Plug x = 27, we get;

(2 × 27 - 3)/3

= 54 - 3/3

= 51/3

= 17

***Since, L.H.S. = R.H.S. hence verified.***

**2.**Solve 0.8 - 0.28x = 1.16 - 0.6x

**Solution:**

0.8 - 0.28x = 1.16 - 0.6x

⇒ 0.6x - 0.28x = 1.16 - 0.8

⇒ 0.32x = 0.36

⇒ x = 0.36/0.32

⇒ x = 36/32

⇒ x = 9/8

Therefore, 9/8 is the required solution.

**Verification:**

**L.H.S.** = 0.8 - 0.28x

Plug x = 9/8, we get;

= 0.8 - 0.28 × 9/8

= 8/10 - 2̶8̶/100 × 9/8̶

= 8/10 - 63/200

= (160 - 63)/200

= 97/200

**Verification:**

**R.H.S.** = 1.16 - 0.6x

= 1.16 - 0.6 × 9/8

= 116/100 - 6̶/10 × 9/8̶

= 116/100 - 27/40

= (232 - 135)/200

= 97/200

***Since, L.H.S. = R.H.S. hence verified.***

### Examples on Solving Linear Equations:

**1.** Solve: (2x + 5)/(x + 4) = 1

**Solution:**

(2x + 5)/(x + 4) = 1

⇒ 2x + 5 = 1(x + 4)

⇒ 2x + 5 = x + 4

⇒ 2x - x = 4 - 5   **(Transferring positive x to the left hand side changes to negative x and again, positive 5 changes to negative 5)**

⇒ x = -1

Therefore, x = - 1 is the required solution of the equation (2x + 5)/(x + 4) = 1

**2.** Solve: 6x - 19 = 3x - 10

**Solution:**

6x - 19 = 3x - 10

⇒ 6x - 3x = - 10 + 19   **(Transferring 3x to L.H.S changes to negative 3x and -19 to R.H.S. changes to positive 19)**

⇒ 3x = 9

⇒ 3x/3 = 9/3   **(Dividing both sides by 3)**

⇒ x = 3

**3.** Solve: 5 - 2(x - 1) = 4(3 - x) - 2x.

**Solution:**

5 - 2(x - 1) = 4(3 - x) - 2x

⇒ 5 - 2x + 2 = 12 - 4x - 2x   **(Removing the brackets and then simplify)**

⇒ 7 - 2x = 12 - 6x   **(Transferring -6x to L.H.S. changes to positive 6x and 7 to R.H.S. changes to negative 7)**

⇒ -2x + 6x = 12 - 7

⇒ 4x = 5

⇒ 4x/4 = 5/4

⇒ x = 5/4

**4.** x/2 + x/3 = x - 7

**Solution:**

x/2 + x/3 = x - 7

Least common multiple of2 and 3 is 6

⇒ (x/2 × 3/3) + (x/3 × 2/2) = x - 7

⇒ 3x/6 + 2x/6 = x - 7

⇒ (3x + 2x)/6 = x - 7

⇒ 5x/6 = x - 7

⇒ 5x = 6(x - 7)

⇒ 5x = 6x - 42   **(Transferring 6x to L.H.S. changes to negative 6x)**

⇒ 5x - 6x = -42

⇒ -x = -42

⇒ x = 42

**5.** 2/(x + 3) = 3/(5 - x)

**Solution:**

2/(x + 3) = 3/(5 - x)

⇒ 3(x + 3) = 2(5 - x)   **(cross multiply and then remove the brackets)**

⇒ 3x + 9 = 10 - 2x   **(Transferring -2x to L.H.S. changes to positive 2x and 9 to R.H.S. changes to -9)**

⇒ 3x + 2x = 10 - 9

⇒ 5x = 1

⇒ 5x/5 = 1/5   **(Dividing both sides by 5)**

⇒ x = 1/5

Solved algebra problems on linear equations in one variable are explained below with the detailed explanation.

Let’s once again recall the methods of solving linear equations in one variable.

● Read the linear problem carefully and note what is given in the question and what is required to find out.

● Denote the unknown by any variable as x, y, ……. (any variable)

● Translate the problem to the language of mathematics or mathematical statements.

● Form the linear equation in one variable using the conditions given in the problems.

● Solve the equation for the unknown.

● Verify to be sure whether the answer satisfies the conditions of the problem.

### Worked-out problems on linear equations in one variable:

**1.** The sum of three consecutive multiples of 4 is 444. Find these multiples.  **Solution:** If x is a multiple of 4, the next multiple is x + 4, next to this is x + 8. Their sum = 444According to the question, x + (x + 4) + (x + 8) = 444 ⇒ x + x + 4 + x + 8 = 444⇒ x + x + x + 4 + 8 = 444 ⇒ 3x + 12 = 444⇒ 3x = 444 - 12 ⇒ x = 432/3 ⇒ x = 144Therefore, x + 4 = 144 + 4 = 148 Therefore, x + 8 - 144 + 8 – 152 **Therefore, the three consecutive multiples of 4 are 144, 148, 152.**

**2.**The denominator of a rational number is greater than its numerator by 3. If the numerator is increased by 7 and the denominator is decreased by 1, the new number becomes 3/2. Find the original number.

**Solution:**

Let the numerator of a rational number = x

Then the denominator of a rational number = x + 3

When numerator is increased by 7, then new numerator = x + 7

When denominator is decreased by 1, then new denominator = x + 3 - 1

The new number formed = 3/2

According to the question,

(x + 7)/(x + 3 - 1) = 3/2

⇒ (x + 7)/(x + 2) = 3/2

⇒ 2(x + 7) = 3(x + 2)

⇒ 2x + 14 = 3x + 6

⇒ 3x - 2x = 14 - 6

⇒ x = 8

**The original number i.e., x/(x + 3) = 8/(8 + 3) = 8/11**

**3.**The sum of the digits of a two digit number is 7. If the number formed by reversing the digits is less than the original number by 27, find the original number.

**Solution:**

Let the units digit of the original number be x.

Then the tens digit of the original number be 7 - x

Then the number formed = 10(7 - x) + x × 1

                                     = 70 - 10x + x = 70 - 9x

On reversing the digits, the number formed

                  = 10 × x + (7 - x) × 1

                  = 10x + 7 - x = 9x + 7

According to the question,

New number = original number - 27

⇒ 9x + 7 = 70 - 9x - 27

⇒ 9x + 7 = 43 - 9x

⇒ 9x + 9x = 43 – 7

⇒ 18x = 36

⇒ x = 36/18

⇒ x = 2

Therefore, 7 - x

              = 7 - 2

              = 5

**The original number is 52**

**4.**A motorboat goes downstream in river and covers a distance between two coastal towns in 5 hours. It covers this distance upstream in 6 hours. If the speed of the stream is 3 km/hr, find the speed of the boat in still water.

**Solution:**

Let the speed of the boat in still water = x km/hr.

Speed of the boat downstream = (x + 3) km/hr.

Time taken to cover the distance = 5 hrs

Therefore, distance covered in 5 hrs = (x + 3) × 5   **(D = Speed × Time)**

Speed of the boat upstream = (x - 3) km/hr

Time taken to cover the distance = 6 hrs.

Therefore, distance covered in 6 hrs = 6(x - 3)

Therefore, the distance between two coastal towns is fixed, i.e., same.

According to the question,

5(x + 3) = 6(x - 3)

⇒ 5x + 15 = 6x - 18

⇒ 5x - 6x = -18 – 15

⇒ -x = -33

⇒ x = 33

**Required speed of the boat is 33 km/hr.**

**5.**Divide 28 into two parts in such a way that 6/5 of one part is equal to 2/3 of the other.

**Solution:**

Let one part be x.

Then other part = 28 - x

It is given 6/5 of one part = 2/3 of the other.

⇒ 6/5x = 2/3(28 - x)

⇒ 3x/5 = 1/3(28 - x)

⇒ 9x = 5(28 - x)

⇒ 9x = 140 - 5x

⇒ 9x + 5x = 140

⇒ 14x = 140

⇒ x = 140/14

⇒ x = 10

**Then the two parts are 10 and 28 - 10 = 18.**

**6.**A total of $10000 is distributed among 150 persons as gift. A gift is either of $50 or $100. Find the number of gifts of each type.

**Solution:**

Total number of gifts = 150

Let the number of $50 is x

Then the number of gifts of $100 is (150 - x)

Amount spent on x gifts of $50 = $ 50x

Amount spent on (150 - x) gifts of $100 = $100(150 - x)

Total amount spent for prizes = $10000

According to the question,

50x + 100 (150 - x) = 10000

⇒ 50x + 15000 - 100x = 10000

⇒ -50x = 10000 - 15000

⇒ -50x = -5000

⇒ x = 5000/50

⇒ x = 100

⇒ 150 - x = 150 - 100 = 50

**Therefore, gifts of $50 are 100 and gifts of $100 are 50.**

The above step-by-step examples demonstrate the solved problems on linear equations in one variable.